Carousel Task – Solutions

Consider the carousel in the picture above. The innermost horse in the picture is 12 feet from the center of the carousel. The outermost horse is 24 feet from the center.

1. Suppose the carousel makes one complete revolution.
   a. Through how many degrees does the outermost horse turn?

   b. Through how many degrees does the innermost horse turn?

   c. Do the two horses travel the same *distance*? Why or why not?

   d. If the two horses travel the same distance, how far do they travel? If they travel different distances, how far does each horse travel? Show how you know.

1.
a. 360 degrees
b. 360 degrees
c. No, the two horses do not travel the same distance. The outside horse travels in a larger circle than the inside horse.
d. In one revolution, the horses travel in one complete circle. Therefore the distance they travel is the circumference of the circle. However, as above, the circles are of different sizes

inside horse (circle of 12 feet radius)
$C=2\pi r = 2\pi(12) = 24\pi \approx 75.4$ ft

outside horse (circle with 24 ft radius)
$C = 2\pi r = 2\pi(24) = 48\pi \approx 150.8$ ft

2. Suppose the carousel rotates through 120°.
   a. Through how many degrees does the outermost horse turn?

   b. Through how many degrees does the innermost horse turn?

   c. How far does each horse travel during this rotation? Show how you know.

2.
a. 120 degrees. Regardless of how far away the horses are from the center, they still rotate by the same amount
b. 120 degrees
c. Even though they rotate by the same amount of degrees, they still travel different distances.

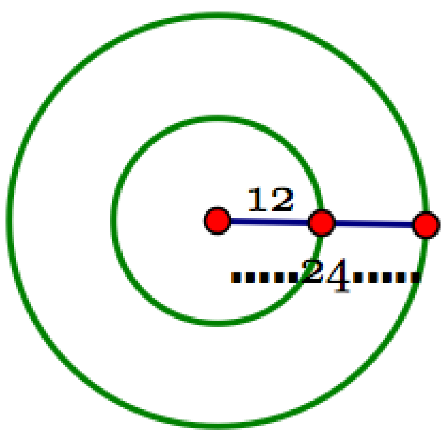
Inside horse
total distance for 360 degrees is 75.4 ft.
but this time they only travel 120 degrees.

120 is 1/3 of 360
so the distance the horse travels in 120 degrees is 1/3 of the distance it travels in 360 degrees
1/3(75.4) ≈ 25.13 ft

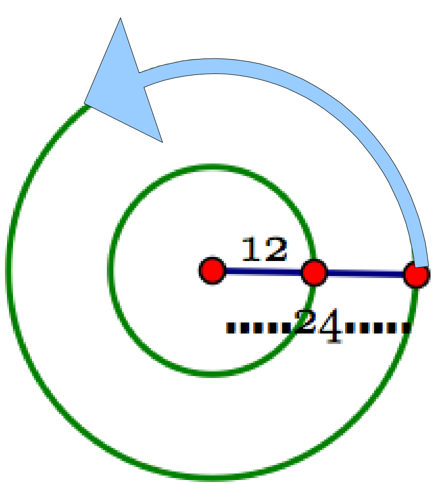
outside horse
total distance for 360 degrees is 150.8 ft.
but this time they only travel 120 degrees.
120 is 1/3 of 360
so the distance the horse travels in 120 degrees is 1/3 of the distance it travels in 360 degrees
1/3(150.8) ≈ 50.27 ft

3. The positions of the innermost and the outermost horses on the carousel can be modeled by two concentric circles. **Concentric** circles are coplanar circles with the same center.
   a. Use your compass to construct concentric circles that represent the positions of the innermost and outermost horses as the carousel rotates.
   b. Consider that the *distance* a horse travels is the *length* of the arc the horse traverses on its circle. Use your diagram and your answers to *Problems 1* and *2* to help you determine a formula for finding the length of any arc on any circle.

3.
a.



b.



When I found the answers for #2, I just took a fraction of the circumference from #1. I took my degrees and divided that my 360 and then multiplied that by the circumference.

$$\frac{x \text{ degrees}}{360 \text{ degrees}} C = \text{Arc length of central angle of x degrees.}$$

In words it is

(fraction of the circle that the arc is taking up) (C)

I could also set it up as a proportion

$$\frac{\text{X degrees}}{\text{360 degrees}} = \frac{\text{Arc length}}{\text{Circumference}}$$

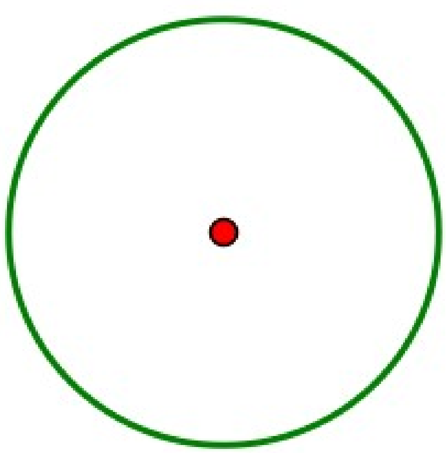
where X is the measure of the central angle and the arc length is the unknown

This proportion could be rearranged or presented in a different yet equivalent way.

$$\frac{\text{X degrees}}{\text{Arc Length}} = \frac{\text{360 degrees}}{\text{Circumference}}$$

The carousel in the picture above needs refurbishing. Suppose, in an effort to make things colorful, the carnival owner wishes to paint a pattern of sectors on the carousel floor. A **sector** of a circle is a region between two radii and an arc of the circle.

4. Consider the floor of the carousel. It can be represented by the outer circle of your diagram in *Problem 3a*. Use your compass to construct a single circle that represents the floor of the carousel. What is the area of the floor? Show how you know?

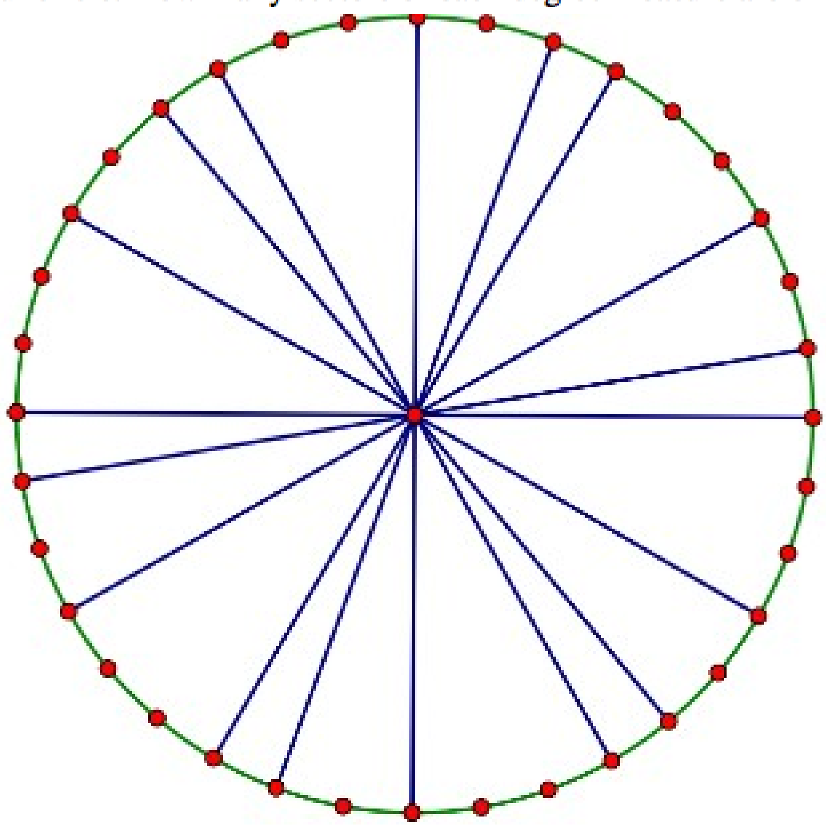


The radius of the outer circle is 24 feet. And the formula to find the area of any circle is $A=\pi r^2$.

So $r=24$

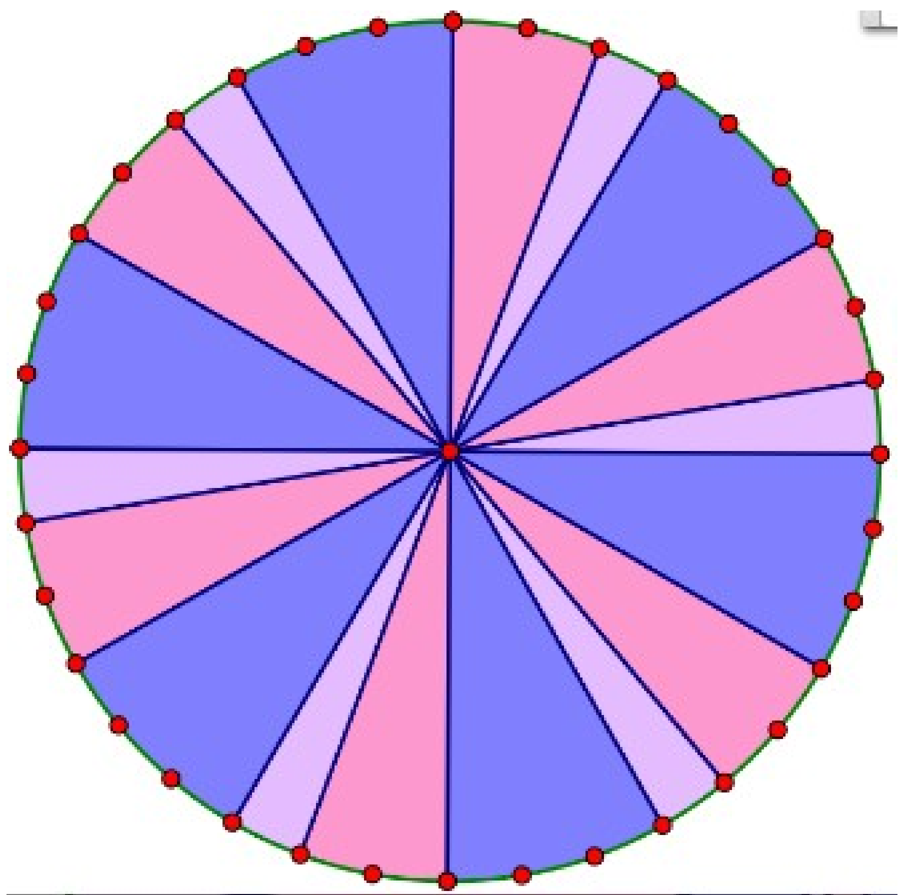
And $A = \pi 24^2 = 576\pi \approx 1809.56$

5. The owner has decided to paint the floor in a repeating pattern of sectors with central angles of 10°, 20°, and then 30°. Use you protractor and a straightedge to draw the pattern on your circle. How many sectors of each degree measure are on your "floor"?



There are 6 sectors of each degree measure on the floor. This makes sense because 6*10 + 6*20 +6(30) = 60 + 120 + 180 = 360 degrees. These had to add up to 360 since that is how many degrees is in a whole circle.

6. Suppose each sector with a central angle of 10° will be painted purple, each sector with a central angle of 20° will be painted pink, and each sector with a central angle of 30° will be painted blue. How many square feet of the floor will be painted purple? pink? blue? Show how you know.



Well we know the whole area is 576π.

So all the areas must be less than that but the sum of the three different colored areas must add up to that at the end.

There are 6 of the 10 degree sectors. So that makes up for 60 degrees of the entire circle.

60 out of 360 is 1/6. So 1/6 of the area is purple. So the area of the purple is (1/6)576π = 96πr

There are also 6 of the 20 degree sectors. That makes up 120 degrees of the circle.

120 out of 360 is 1/3. So 1/3 of the area is pink. So the area of the pink is (1/3)576π = 192π

And again there are 6 of the 30 degree sectors. So that is 180 degrees of the circle.

180 out of 360 is 1/2. So 1/2 of the area is blue. So the area of the blue is (1/2)576π = 288π

7. Use what you have learned in *Problems 4 – 6* to help you determine a formula for finding the area of any sector of any circle.

So the area of any sector of any circle can be found by taking the fraction of circle times the whole area.

$$\text{Area of sector} = \frac{\text{X degrees}}{\text{360 degrees}} A$$

where X degrees represents the measure of the central angle of the sector and A represents the area of the whole circle

Again, this can be written as a proportion.

$$\frac{\text{Area of Sector}}{\text{Area of Circle}} = \frac{\text{X degrees}}{\text{360 Degrees}}$$

Where X is the measure of the central angle of the sector in degrees and the area of the sector is the unknown.